Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC2

Unit Pure Core 2

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

A

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 A geometric series has first term 80 and common ratio $\frac{1}{2}$.

(a) Find the third term of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 12 terms of the series, giving your answer to two decimal places. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

a) $a = 80 \qquad r = \frac{1}{2}$

$U_3 = ar^2$

$\therefore U_3 = 80 \times (0.5)^2$

$U_3 = 20$

b) $S_\infty = \frac{a}{1-r}$

$\therefore S_\infty = \frac{80}{1-0.5}$

$S_\infty = 160$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

c) $S_{12} = \dfrac{80(0.5^{12}-1)}{0.5-1}$

$S_{12} = 159.96$ to 2dp.

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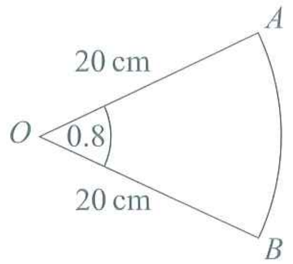
2 The diagram shows a sector OAB of a circle with centre O.

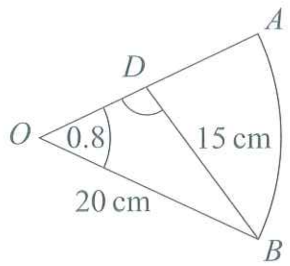


The radius of the circle is 20 cm and the angle $AOB = 0.8$ radians.

(a) Find the length of the arc AB. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) A line from B meets the radius OA at the point D, as shown in the diagram below.



The length of BD is 15 cm. Find the size of the **obtuse** angle ODB, in **radians**, giving your answer to three significant figures. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

a) arc $AB = \theta r$

$= 0.8 \times \cancel{16}\ 20$

$= 16$ cm

b) Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 20^2 \times 0.8$

$= 160$ cm^2

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

c)
$$\frac{\sin \angle OPB}{20} = \frac{\sin 0.8^c}{15}$$

$$\sin \angle OPB = \frac{20 \sin 0.8^c}{15}$$

$\sin \angle OPB = 0.9564747879$

$\angle ODB = 1.27^c$

$\sin \theta = \sin(\pi - \theta)$

$\therefore \sin 1.27^c = \sin 1.87^c$

→ as angle ODB is obtuse it is equal to 1.87^c to 3 sf.

Do not write outside the box

3 (a) (i) Using the binomial expansion, or otherwise, express $(2+y)^3$ in the form $a + by + cy^2 + y^3$, where a, b and c are integers. *(2 marks)*

(ii) Hence show that $(2+x^{-2})^3 + (2-x^{-2})^3$ can be expressed in the form $p + qx^{-4}$, where p and q are integers. *(3 marks)*

(b) (i) Hence find $\int \left[(2+x^{-2})^3 + (2-x^{-2})^3\right] \mathrm{d}x$. *(2 marks)*

(ii) Hence find the value of $\int_1^2 \left[(2+x^{-2})^3 + (2-x^{-2})^3\right] \mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE	Answer space for question 3
a)i)	$(2+y)^3 = (2)^3 + 3(2)^2(y)^1 + 3(2)^1(y)^2 + (y)^3$
	$= 8 + 12y + 6y^2 + y^3$
ii)	$(2+x^{-2})^3 = 8 + 12x^{-2} + 6x^{-4} + x^{-6}$
	$(2-x^{-2})^3 = 8 - 12x^{-2} + 6x^{-4} - x^{-6}$
	$(8 + 12x^{-2} + 6x^{-4} + x^{-6})$
	$+ (8 - 12x^{-2} + 6x^{-4} - x^{-6})$
	$= 16 + 12x^{-4}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

b) i) $\int (16 + 12x^{-4}) \, dx$

$= 16x - 4x^{-3} + c$

ii) $\left[16x - \frac{4}{x^3} \right]_1^2$

$= \left(32 - \frac{4}{8}\right) - \left(16 - \frac{4}{1}\right)$

$= 19.5$

4 (a) Sketch the graph of $y = 9^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

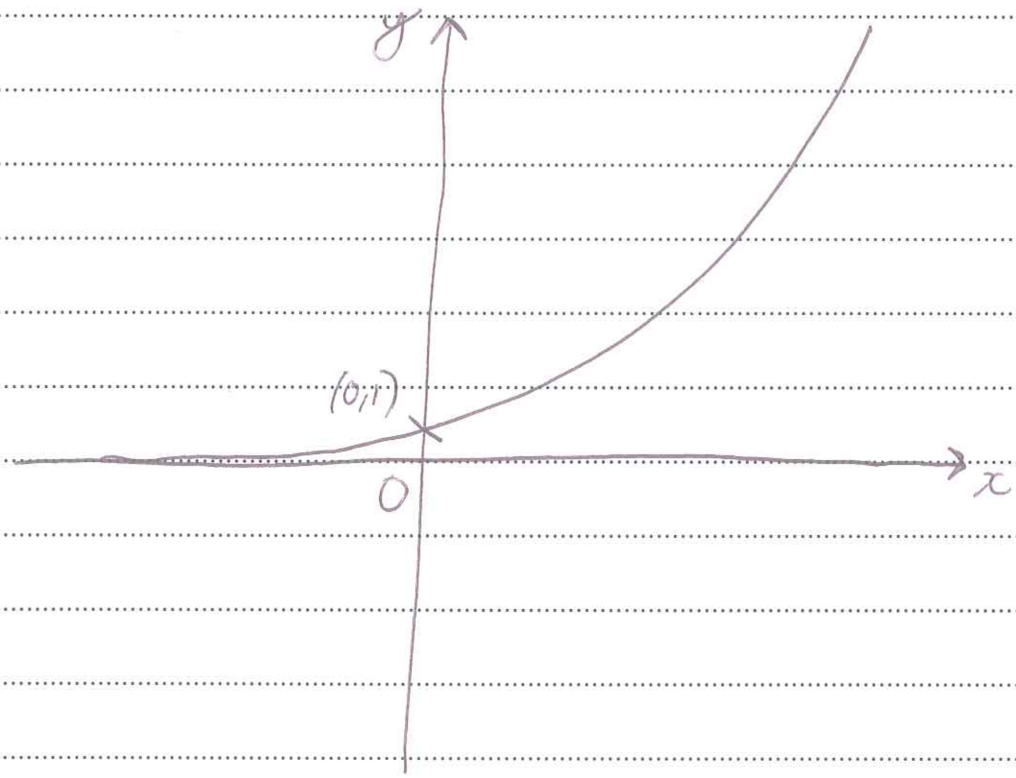
(b) Use logarithms to solve the equation $9^x = 15$, giving your value of x to three significant figures. *(2 marks)*

(c) The curve $y = 9^x$ is reflected in the y-axis to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

a)



b) $9^x = 15$

$\log_9 15 = x$

$x = 1.23$ to 3sf

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

c) $y = 9^{x} \longrightarrow y = 9^{-x}$

$f(x) = 9^{-x}$

Do not write outside the box

5 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 \sqrt{8x^3+1}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Describe the single transformation that maps the graph of $y = \sqrt{8x^3+1}$ onto the graph of $y = \sqrt{x^3+1}$. *(2 marks)*

(c) The curve with equation $y = \sqrt{x^3+1}$ is translated by $\begin{bmatrix} 2 \\ -0.7 \end{bmatrix}$ to give the curve with equation $y = \mathrm{g}(x)$. Find the value of $\mathrm{g}(4)$. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

a) $\int_0^2 \sqrt{8x^3+1}\, dx \qquad h = 0.5$

$x=0 \quad x=0.5 \quad x=1 \quad x=1.5 \quad x=2$

$y=1 \quad y=\sqrt{2} \quad y=3 \quad y=\sqrt{28} \quad y=\sqrt{65}$

Area $\approx \frac{1}{2}h$ (exterior ordinates $+ 2\sum$ interior ordinates)

$\therefore$ Area $\approx \frac{1}{2} \times 0.5 \left[1+\sqrt{65} + 2(\sqrt{2}+3+\sqrt{28})\right]$

$\rightarrow$ Area ≈ 7.12 to 3sf

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

b) $\sqrt{8x^3+1}$ $\sqrt{x^3+1}$

$\downarrow$

$\sqrt{(2x)^3+1}$ becomes $\sqrt{(x)^3+1}$

~~x~~ $(2x)$ has been replaced with (x)

$\therefore$ stretch in the x-direction by scale factor 2.

c) $y=\sqrt{x^3+1}$ translated $\begin{bmatrix} 2 \\ -0.7 \end{bmatrix}$

$\therefore$ replace (x) with $(x-2)$ and (y) with $(y+0.7)$

$\rightarrow$ $y+0.7=\sqrt{(x-2)^3+1}$

$\therefore$ $y=\sqrt{(x-2)^3+1}-0.7$

$y(x)=\sqrt{(x-2)^3+1}-0.7$

$\therefore$ $y(4)=\sqrt{(4-2)^3+1}-0.7$

$y(4)=2.3$

Turn over ▶

Do not write outside the box

6 A curve has the equation

$$y = \frac{12 + x^2\sqrt{x}}{x}, \quad x > 0$$

(a) Express $\dfrac{12 + x^2\sqrt{x}}{x}$ in the form $12x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the normal to the curve at the point on the curve where $x = 4$. *(4 marks)*

(iii) The curve has a stationary point P. Show that the x-coordinate of P can be written in the form 2^k, where k is a rational number. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 6

a) $\dfrac{12 + x^2\sqrt{x}}{x}$ $\quad x^2\sqrt{x} = x^{5/2}$

$\rightarrow \dfrac{12 + x^{5/2}}{x} = 12x^{-1} + x^{3/2}$

b)i) $\dfrac{dy}{dx} = -12x^{-2} + \dfrac{3}{2}x^{\frac{1}{2}}$

ii) $\dfrac{dy}{dx}$ when $x = 4$ is $\dfrac{3\sqrt{4}}{2} - \dfrac{12}{(2)^2}$

$= 3 - \dfrac{3}{4}$

$= 9/4$

$\rightarrow$ gradient of tangent when $x = 4$ is $\dfrac{9}{4}$

$\therefore$ gradient of normal is $-\dfrac{4}{9}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

when $x=4$ $\quad y = \frac{12}{4} + (4)^{3/2}$

$y = 3 + 8$

$y = 11$

$y = -\frac{4}{9}x + c \qquad (\underset{x}{4}, \underset{y}{11})$

$\rightarrow 11 = \left(-\frac{4}{9} \times 4\right) + c$

$11 = -\frac{16}{9} + c$

$c = \frac{115}{9}$

$\rightarrow y = -\frac{4}{9}x + \frac{115}{9}$

$4x + 9y = 115$

iii) at P $\frac{dy}{dx} = 0 \quad \therefore \frac{3\sqrt{x}}{2} - \frac{12}{x^2} = 0$

$\rightarrow \frac{3\sqrt{x}}{2} = \frac{12}{x^2}$

$3x^{5/2} = 24$

$x^{5/2} = 8$

$x^5 = 8^2$

$x^5 = 64 = 2^6$

$\rightarrow x^5 = 2^6$

$\therefore (x^5)^{1/5} = (2^6)^{1/5} \rightarrow x = 2^{6/5}$

Turn over ▶

Do not write outside the box

7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 96$ and $u_2 = 72$.

The limit of u_n as n tends to infinity is 24.

(a) Show that $p = \frac{2}{3}$. *(4 marks)*

(b) Find the value of u_3. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

a) $U_2 = pU_1 + q \qquad U_2 = 72 \qquad U_1 = 96$

$\therefore 72 = 96p + q$

$L = pL + q \qquad L = 24$

$\therefore 24 = 24p + q$

$(1)\ 96p + q = 72$

$-\ (2)\ 24p + q = 24$

$72p = 48$

$p = \frac{48}{72} \rightarrow \underline{p = \frac{2}{3}}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

b) $96p + q = 72$ $\qquad p = \frac{2}{3}$

$\therefore \left(96 \times \frac{2}{3}\right) + q = 72$

$64 + q = 72$

$q = 8$

$U_3 = pU_2 + q$ $\qquad U_2 = 72$

$\therefore U_3 = \left(\frac{2}{3} \times 72\right) + 8$

$U_3 = 48 + 8$

$\underline{U_3 = 56}$

8 (a) Given that $\log_a b = c$, express b in terms of a and c. *(1 mark)*

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2\log_2(x+7) - \log_2(x+5) = 3$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 8

a) $\log_a b = c \rightarrow b = a^c$

b) $2\log_2(x+7) - \log_2(x+5) = 3$

$\log_2(x+7)^2 - \log_2(x+5) = \log_2 8$

$\log_2(x^2+14x+49) = \log_2 8 + \log_2(x+5)$

$\log_2(x^2+14x+49) = \log_2 8(x+5)$

$\rightarrow x^2+14x+49 = 8x+40$

$x^2+6x+9=0$

$(x+3)^2=0$

$\rightarrow \underline{x=-3}$

Do not write outside the box

9 (a) (i) On the axes given below, sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(ii) Solve the equation $\tan x = -1$, giving all values of x in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

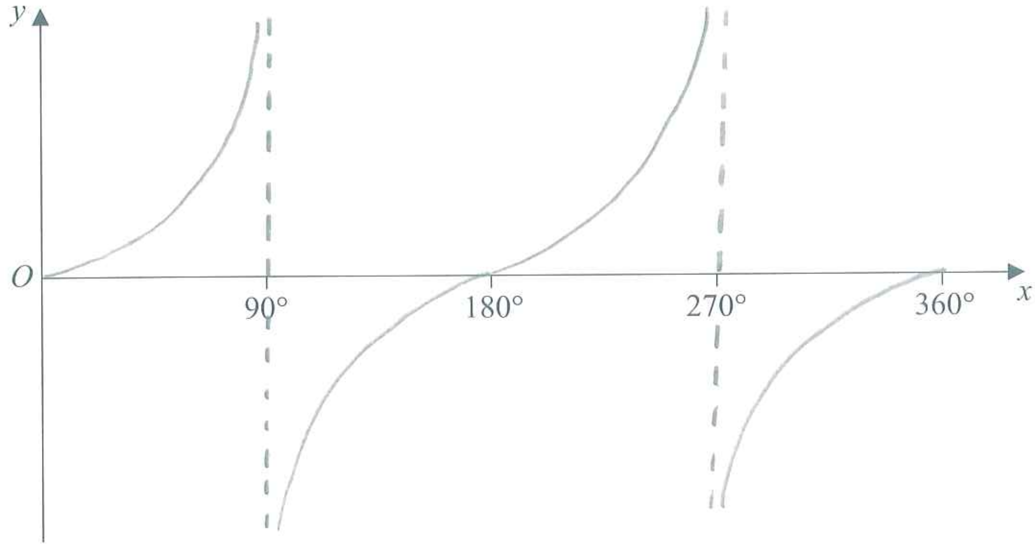
(b) (i) Given that $6 \tan\theta \sin\theta = 5$, show that $6\cos^2\theta + 5\cos\theta - 6 = 0$. *(3 marks)*

(ii) **Hence** solve the equation $6\tan 3x \sin 3x = 5$, giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 9

(a)(i)



a)ii) $\tan x = -1$

$x = \tan^{-1}(-1)$

$x = -45° + 180n°$

Range is $0° \leqslant x \leqslant 360°$

$\therefore x = 135°, 315°$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 9

b) i) $6 \tan\theta \sin\theta = 5$ $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

$6\left(\frac{\sin\theta \sin\theta}{\cos\theta}\right) = 5$

$6\sin^2\theta = 5\cos\theta$ $\boxed{\sin^2\theta = 1-\cos^2\theta}$

$6(1-\cos^2\theta) = 5\cos\theta$

$6 - 6\cos^2\theta = 5\cos\theta$

$\rightarrow 6\cos^2\theta + 5\cos\theta - 6 = 0$

ii) $6\tan 3x \sin 3x = 5$

$\rightarrow 6\cos^2 3x + 5\cos 3x - 6 = 0$

FACTORISE

sum $= 5$ $\quad 9-4$

product $= -36$ $\quad 9\times -4$

	$2\cos 3x$	$+3$
$3\cos 3x$	$6\cos^2 3x$	$9\cos 3x$
-2	$-4\cos 3x$	-6

$(3\cos 3x - 2)(2\cos 3x + 3) = 0$

PTO

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QUESTION PART REFERENCE

Answer space for question 9

① $3\cos 3x - 2 = 0$
$3\cos 3x = 2$
$\cos 3x = \frac{2}{3}$

② $2\cos 3x + 3 = 0$
$2\cos 3x = -3$
$\cos 3x = -\frac{3}{2}$

$-1 \leq \cos 3x \leq 1 \quad \therefore \cos 3x \neq -\frac{3}{2}$

hence $\cos 3x = \frac{2}{3}$ is the only possible solution

$\cos 3x = \frac{2}{3}$
$3x = \pm 48.1896851^\circ + 360n^\circ$
$x = \pm 16.06^\circ + 120n^\circ$

Range is $0 \leq x \leq 180^\circ$

$\therefore x = 16^\circ, 104^\circ, 136^\circ$ to the nearest degree.

END OF QUESTIONS