

WARNING	Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE .		
PAPER – II MATHEMATICS-2015			
Version Code	B1	Question Booklet Serial Number :	2138662
Time : 150 Minutes	Number of Questions : 120		Maximum Marks : 480
Name of Candidate			
Roll Number			
Signature of Candidate			
INSTRUCTIONS TO THE CANDIDATE			
<ol style="list-style-type: none"> 1. Please ensure that the VERSION CODE shown at the top of this Question Booklet is the same as that shown in the Admit card issued to you. If you have received a Question Booklet with a different Version Code, please get it replaced with a Question Booklet with the same Version Code as that of the Admit card. THIS IS VERY IMPORTANT. 2. Please fill in the items such as name, roll number and signature in the columns given above. Please also write Question Booklet Sl. No. given at the top of this page against item 3 in the OMR Answer Sheet. 3. This Question Booklet contains 120 questions. For each question, five answers are suggested and given against (A), (B), (C), (D) and (E) of which only one will be the Most Appropriate Answer. Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either Blue or Black ball-point pen only. 4. Negative Marking: In order to discourage wild guessing, the score will be subjected to penalization formula based on the number of right answers actually marked and the number of wrong answers marked. Each correct answer will be awarded FOUR marks. ONE mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked. 5. Please read the instructions given in the OMR Answer Sheet for marking answers. Candidates are advised to strictly follow the instructions contained in the OMR Answer Sheet. 			
IMMEDIATELY AFTER OPENING THIS QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET ISSUED CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.			
DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.			

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**PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS
120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120.
PRINTED PAGES : 32**

1. The domain of the function $f(x) = \sqrt{7-3x} + \log_e x$ is
 (A) $0 < x < \infty$ (B) $\frac{7}{3} \leq x < \infty$ (C) $0 < x \leq \frac{7}{3}$
 (D) $-\infty < x < 0$ (E) $-\infty < x \leq \frac{7}{3}$

2. If $f(1) = 1$, $f(2n) = f(n)$ and $f(2n+1) = (f(n))^2 - 2$ for $n = 1, 2, 3, \dots$, then the value of $f(1) + f(2) + \dots + f(25)$ is equal to
 (A) 1 (B) -15 (C) -17 (D) -1 (E) 13

3. Let X and Y be two non-empty sets such that $X \cap A = Y \cap A = \emptyset$ and $X \cup A = Y \cup A$ for some non-empty set A . Then
 (A) X is a proper subset of Y
 (B) Y is a proper subset of X
 (C) $X = Y$
 (D) X and Y are disjoint sets
 (E) $X \setminus A = \emptyset$

Space for rough work

4. The set $(A \setminus B) \cup (B \setminus A)$ is equal to
- (A) $[A \setminus (A \cap B)] \cap [B \setminus (A \cap B)]$ (B) $(A \cup B) \setminus (A \cap B)$
 (C) $A \setminus (A \cap B)$ (D) $\overline{A \cap B} \setminus A \cup B$
 (E) $\overline{(A \cup B)} \setminus \overline{(A \cup B)}$
5. The range of the function $f(x) = \frac{1}{2 - \cos 3x}$ is
- (A) $(-2, \infty)$ (B) $[-2, 3]$ (C) $\left(\frac{1}{3}, 2\right)$ (D) $\left(\frac{1}{2}, 1\right)$ (E) $\left[\frac{1}{3}, 1\right]$
6. In a class of 80 students numbered 1 to 80, all odd numbered students opt for Cricket, students whose numbers are divisible by 5 opt for Football and those whose numbers are divisible by 7 opt for Hockey. The number of students who do not opt any of the three games, is
- (A) 13 (B) 24 (C) 28 (D) 52 (E) 67
7. If $\operatorname{Re}(1 + iy)^3 = -26$, where y is a real number, then the value of $|y|$ is
- (A) 2 (B) 3 (C) 4 (D) 6 (E) 9
8. If $z = x + iy$ is a complex number such that $|z| = \operatorname{Re}(iz) + 1$, then the locus of z is
- (A) $x^2 + y^2 = 1$ (B) $x^2 = 2y - 1$ (C) $y^2 = 2x - 1$
 (D) $y^2 = 1 - 2x$ (E) $x^2 = 1 - 2y$

Space for rough work

13. Sum of the roots of the equation $|x-3|^2 + |x-3| - 2 = 0$ is equal to
 (A) 2 (B) 4 (C) 6 (D) 16 (E) -2
14. The quadratic equation whose roots are three times the roots of the equation $2x^2 + 3x + 5 = 0$, is
 (A) $2x^2 + 9x + 45 = 0$ (B) $2x^2 + 9x - 45 = 0$
 (C) $5x^2 + 9x + 45 = 0$ (D) $2x^2 - 9x + 45 = 0$
 (E) $2x^2 + 9x + 49 = 0$
15. If x is real number, then $\frac{x}{x^2 - 5x + 9}$ must lie between
 (A) $\frac{1}{11}$ and 1 (B) -1 and $\frac{1}{11}$ (C) -11 and 1
 (D) -1 and 11 (E) $-\frac{1}{11}$ and 1
16. If the roots of the equation $(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$ are equal, then $a^2 + b^2 + c^2 =$
 (A) $a + b + c$ (B) $2a + b + c$ (C) $3abc$
 (D) $ab + bc + ca$ (E) abc

Space for rough work

9. Let $i^2 = -1$. Then $\left(i^{10} - \frac{1}{i^{11}}\right) + \left(i^{11} - \frac{1}{i^{12}}\right) + \left(i^{12} - \frac{1}{i^{13}}\right) + \left(i^{13} - \frac{1}{i^{14}}\right) + \left(i^{14} + \frac{1}{i^{15}}\right) =$
 (A) $-1+i$ (B) $-1-i$ (C) $1+i$ (D) $-i$ (E) i
10. If $f(z) = \frac{1-z^3}{1-z}$, where $z = x+iy$ with $z \neq 1$, then $\operatorname{Re}\{\overline{f(z)}\} = 0$ reduces to
 (A) $x^2 + y^2 + x + 1 = 0$ (B) $x^2 - y^2 + x - 1 = 0$ (C) $x^2 - y^2 - x + 1 = 0$
 (D) $x^2 - y^2 + x + 1 = 0$ (E) $x^2 - y^2 + x + 2 = 0$
11. If $z = 1+i$, then the argument of $z^2 e^{z-i}$ is
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) 0
12. If the difference between the roots of $x^2 + 2px + q = 0$ is two times the difference between the roots of $x^2 + qx + \frac{p}{4} = 0$, where $p \neq q$, then
 (A) $p - q + 1 = 0$ (B) $p - q - 1 = 0$ (C) $p + q - 1 = 0$
 (D) $p + q + 1 = 0$ (E) $q - 4p + 1 = 0$

 Space for rough work

17. If one root of a quadratic equation is $\frac{1}{1+\sqrt{3}}$, then the quadratic equation is
- (A) $2x^2 + x - 1 = 0$ (B) $2x^2 - 2x - 1 = 0$ (C) $2x^2 + 2x + 1 = 0$
(D) $2x^2 + x + 1 = 0$ (E) $2x^2 + 2x - 1 = 0$
18. The 5th and 8th terms of a G.P. are 1458 and 54 respectively. The common ratio of the G.P. is
- (A) $\frac{1}{3}$ (B) 3 (C) 9
(D) $\frac{1}{9}$ (E) $\frac{1}{8}$
19. Let $S(n)$ denote the sum of the digits of a positive integer n . For example,
 $S(178) = 1 + 7 + 8 = 16$. Then the value of $S(1) + S(2) + S(3) + \dots + S(99)$ is equal to
- (A) 476 (B) 998 (C) 782 (D) 900 (E) 855

Space for rough work

20. An A.P. consists of 23 terms. If the sum of the 3 terms in the middle is 141 and the sum of the last 3 terms is 261, then the first term is
 (A) 6 (B) 5 (C) 4 (D) 3 (E) 2
21. If 4th term of a G.P. is 32 whose common ratio is half of the first term, then the 15th term is
 (A) 2^{12} (B) 2^{18} (C) 2^{14} (D) 2^{16} (E) 2^{10}
22. Let S_1 be a square of side 5 cm. Another square S_2 is drawn by joining the midpoints of the sides of S_1 . Square S_3 is drawn by joining the midpoints of the sides of S_2 and so on. Then (area of S_1 + area of S_2 + area of S_3 + ... + area of S_{10}) =
 (A) $25\left(1 - \frac{1}{2^{10}}\right)$ (B) $50\left(1 - \frac{1}{2^{10}}\right)$ (C) $2\left(1 - \frac{1}{2^{10}}\right)$
 (D) $1 - \frac{1}{2^{10}}$ (E) $10\left(1 - \frac{1}{2^{10}}\right)$
23. If a, b, c are in A.P. and if their squares taken in the same order form a G.P., then $(a+c)^4 =$
 (A) $16a^2c^2$ (B) $4a^2c^2$ (C) $8a^2c^2$ (D) $2a^2c^2$ (E) a^2c^2

Space for rough work

24. The value of x satisfying the relation $11({}^xC_3) = 24({}^{(x+1)}C_2)$ is
 (A) 8 (B) 9 (C) 11 (D) 10 (E) 12
25. The power of x in the term with the greatest coefficient in the expansion of $\left(1 + \frac{x}{2}\right)^{10}$ is
 (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
26. The number of 5-digit numbers (no digit is repeated) that can be formed by using the digits 0, 1, 2, ..., 7 is
 (A) 1340 (B) 1860 (C) 2340 (D) 2160 (E) 3200
27. If m_1 and m_2 satisfy the relation ${}^{m+5}P_{m+1} = \frac{11}{2}(m-1)({}^{m+3}P_m)$, then $m_1 + m_2$ is equal to
 (A) 10 (B) 9 (C) 13 (D) 17 (E) 15
28. Sum of coefficients of the last 6 terms in the expansion of $(1+x)^{11}$ when the expansion is in ascending powers of x , is
 (A) 2048 (B) 32 (C) 512 (D) 64 (E) 1024

Space for rough work

29. If $C_0, C_1, C_2, \dots, C_{15}$ are binomial coefficients in $(1+x)^{15}$, then

$$\frac{C_1}{C_0} + 2 \frac{C_2}{C_1} + 3 \frac{C_3}{C_2} + \dots + 15 \frac{C_{15}}{C_{14}} =$$

- (A) 60 (B) 120 (C) 64 (D) 124 (E) 144

30. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 6 & 12 \end{vmatrix}$ and $\Delta' = \begin{vmatrix} 4 & 8 & 15 \\ 3 & 6 & 12 \\ 2 & 3 & 5 \end{vmatrix}$, then

- (A) $\Delta' = 2\Delta$ (B) $\Delta' = -2\Delta$ (C) $\Delta' = \Delta$ (D) $\Delta' = -\Delta$ (E) $\Delta' = 3\Delta$

31. The roots of the equation $\begin{vmatrix} 1+x & 3 & 5 \\ 2 & 2+x & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$ are

- (A) 2, 1, -9 (B) 1, 1, -9 (C) -1, 1, -9 (D) -2, -1, -8 (E) -2, 1, 1

32. If $\begin{pmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{pmatrix}$ is singular, then the value of b is equal to

- (A) -3 (B) 3 (C) -6 (D) 6 (E) -2

Space for rough work

33. If $A = \begin{bmatrix} x & x-1 \\ 2x & 1 \end{bmatrix}$ and if $\det A = -9$, then the values of x are

- (A) $\frac{3}{2}, -3$ (B) $\frac{-2}{3}, 3$ (C) $\frac{2}{3}, 3$ (D) $\frac{-3}{2}, 3$ (E) $\frac{-3}{2}, -3$

34. The value of the determinant $\begin{vmatrix} \cos^2 54^\circ & \cos^2 36^\circ & \cot 135^\circ \\ \sin^2 53^\circ & \cot 135^\circ & \sin^2 37^\circ \\ \cot 135^\circ & \cos^2 25^\circ & \cos^2 65^\circ \end{vmatrix}$ is equal to

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

35. If A and B are square matrices of the same order and if $A = A^T$, $B = B^T$, then

$$(ABA)^T =$$

- (A) BAB (B) ABA (C) $ABAB$
(D) AB^T (E) $(AB)^T$

36. If $|x-3| < 2x+9$, then x lies in the interval

- (A) $(-\infty, -2)$ (B) $(-2, 0)$ (C) $(-2, \infty)$
(D) $(2, \infty)$ (E) $(-12, -2)$

Space for rough work

37. The area and perimeter of a rectangle are A and P respectively. Then P and A satisfy the inequality
- (A) $P + A > PA$ (B) $P^2 \leq A$ (C) $A - P < 2$
 (D) $P^2 \leq 4A$ (E) $P^2 \geq 16A$
38. For any two statements p and q , the statement $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to
- (A) $\sim p$ (B) p (C) q (D) $\sim q$ (E) $p \vee q$
39. Let p, q, r be three statements. Then $\sim(p \vee (q \wedge r))$ is equal to
- (A) $(\sim p \wedge \sim q) \wedge (\sim p \wedge \sim r)$ (B) $(\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$
 (C) $(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ (D) $(\sim p \vee \sim q) \vee (\sim p \wedge \sim r)$
 (E) $(\sim p \wedge \sim q) \vee (\sim p \vee \sim r)$
40. Consider the two statements P : He is intelligent and Q : He is strong. Then the symbolic form of the statement "It is not true that he is either intelligent or strong" is
- (A) $\sim P \vee Q$ (B) $\sim P \vee \sim Q$ (C) $\sim P \wedge Q$ (D) $P \vee \sim Q$ (E) $\sim(P \vee Q)$

Space for rough work

41. If $\tan \frac{\theta}{2} = \frac{1}{2}$, then the value of $\sin \theta$ is
- (A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) 1 (E) $\frac{2}{5}$
42. If $x = 5 + 2 \sec \theta$ and $y = 5 + 2 \tan \theta$, then $(x-5)^2 - (y-5)^2$ is equal to
- (A) 3 (B) 1 (C) 0 (D) 4 (E) 2
43. The value of $\tan 15^\circ + \tan 75^\circ$ is equal to
- (A) $2\sqrt{3}$ (B) 2 (C) $2 - \sqrt{3}$ (D) $4\sqrt{3}$ (E) 4
44. The period of the function $f(x) = \cos 4x + \tan 3x$ is
- (A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{2}$ (D) π (E) 2π

Space for rough work

45. If $\sin(\theta + \phi) = n \sin(\theta - \phi)$, $n \neq 1$, then the value of $\frac{\tan \theta}{\tan \phi}$ is equal to

- (A) $\frac{n}{n-1}$ (B) $\frac{n+1}{n-1}$ (C) $\frac{n}{1-n}$ (D) $\frac{n-1}{n+1}$ (E) $\frac{1+n}{1-n}$

46. $2 \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) =$

- (A) $\tan^{-1}\left(\frac{16}{13}\right)$ (B) $\tan^{-1}\left(\frac{17}{23}\right)$ (C) $\frac{\pi}{4}$ (D) 0 (E) $\tan^{-1}\left(\frac{11}{12}\right)$

47. If $\cos^{-1} x + \cos^{-1} y = \frac{2\pi}{7}$, then the value of $\sin^{-1} x + \sin^{-1} y$ is equal to

- (A) $\frac{4\pi}{7}$ (B) $\frac{3\pi}{7}$ (C) $\frac{2\pi}{7}$ (D) $\frac{6\pi}{7}$ (E) $\frac{5\pi}{7}$

48. If $\cos^{-1} x > \sin^{-1} x$, then x lies in the interval

- (A) $\left[\frac{1}{2}, 1\right]$ (B) $(0, 1]$ (C) $\left[-1, \frac{1}{\sqrt{2}}\right)$ (D) $[-1, 1]$ (E) $[0, 1]$

49. If $0 \leq x \leq 2\pi$, then the number of solutions of the equation $\sin^8 x + \cos^6 x = 1$ is

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 8

Space for rough work

50. Let $A(0,0)$ and $B(8,0)$ be two vertices of a right angled triangle whose hypotenuse is BC . If the circumcentre is $(4, 2)$, then the point C is
 (A) $(2, 4)$ (B) $(0, 8)$ (C) $(0, 4)$ (D) $(0, 6)$ (E) $(2, 8)$
51. If coordinates of the circumcentre and the orthocentre of a triangle are respectively $(5, 5)$ and $(2, 2)$, then the coordinates of the centroid are
 (A) $(1, 1)$ (B) $(3, 1)$ (C) $(3, 3)$ (D) $(2, 2)$ (E) $(4, 4)$
52. If the points $A(3, 4)$, $B(x_1, y_1)$ and $C(x_2, y_2)$ are such that both $3, x_1, x_2$ and $4, y_1, y_2$ are in A.P., then
 (A) A, B, C are vertices of an isosceles triangle
 (B) A, B, C are collinear points
 (C) A, B, C are vertices of a right angled triangle
 (D) A, B, C are vertices of a scalene triangle
 (E) A, B, C are vertices of an equilateral triangle
53. If p_1 and p_2 are respectively length of perpendiculars from the origin to the straight lines $x \sec \theta + y \operatorname{cosec} \theta = a$ and $x \cos \theta - y \sin \theta = a \cos 2\theta$, then $4p_1^2 + p_2^2 =$
 (A) 1 (B) a^2 (C) $\frac{1}{a^2}$ (D) a (E) $\frac{1}{a}$

Space for rough work

54. The distance between the point $(1, 2)$ and the point of intersection of the lines $2x + y = 2$ and $x + 2y = 2$ is
- (A) $\frac{\sqrt{17}}{3}$ (B) $\frac{\sqrt{16}}{3}$ (C) $\frac{\sqrt{17}}{5}$ (D) $\frac{\sqrt{19}}{3}$ (E) $\frac{\sqrt{19}}{5}$
55. If a straight line is perpendicular to $2x + 8y = 10$ and meets the x -axis at $(5, 0)$, then it meets the y -axis at
- (A) $(0, -2)$ (B) $(0, -8)$ (C) $(0, -10)$ (D) $(0, -16)$ (E) $(0, -20)$
56. If the straight line $5x + y = k$ forms a triangle with the coordinate axes of area 10 sq. units, then the values of k are
- (A) ± 15 (B) ± 10 (C) ± 5 (D) ± 20 (E) ± 7
57. A circle passes through the point $(6, 2)$. If segments of the straight lines $x + y = 6$ and $x + 2y = 4$ are two diameters of the circle, then its radius is
- (A) 4 (B) 8 (C) $\sqrt{5}$ (D) $2\sqrt{5}$ (E) $4\sqrt{5}$
58. The parametric form of equation of the circle $x^2 + y^2 - 6x + 2y - 28 = 0$ is
- (A) $x = -3 + \sqrt{38} \cos \theta, y = -1 + \sqrt{38} \sin \theta$ (B) $x = \sqrt{28} \cos \theta, y = \sqrt{28} \sin \theta$
 (C) $x = -3 - \sqrt{38} \cos \theta, y = 1 + \sqrt{38} \sin \theta$ (D) $x = 3 + \sqrt{38} \cos \theta, y = -1 + \sqrt{38} \sin \theta$
 (E) $x = 3 + 38 \cos \theta, y = -1 + 38 \sin \theta$

Space for rough work

59. The point on the circle $(x-1)^2 + (y-1)^2 = 1$ which is nearest to the circle $(x-5)^2 + (y-5)^2 = 4$ is
- (A) $(2, 2)$ (B) $\left(\frac{3}{2}, \frac{3}{2}\right)$ (C) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}, \frac{\sqrt{2}+1}{\sqrt{2}}\right)$
 (D) $(\sqrt{2}, \sqrt{2})$ (E) $(1+\sqrt{2}, 1+\sqrt{2})$
60. The line segment joining $(5, 0)$ and $(10 \cos \theta, 10 \sin \theta)$ is divided internally in the ratio $2:3$ at P . If θ varies, then the locus of P is
- (A) a pair of straight lines (B) a circle (C) a straight line
 (D) a parabola (E) an ellipse
61. $ABCD$ is a square with side a . If AB and AD are along the coordinate axes, then the equation of the circle passing through the vertices A, B and D is
- (A) $x^2 + y^2 = \sqrt{2}a(x+y)$ (B) $x^2 + y^2 = \frac{a}{\sqrt{2}}(x+y)$ (C) $x^2 + y^2 = a(x+y)$
 (D) $x^2 + y^2 = a^2(x+y)$ (E) $a(x^2 + y^2) = x+y$
62. The distance between the directrices of the hyperbola $x^2 - y^2 = 9$ is
- (A) $\frac{9}{\sqrt{2}}$ (B) $\frac{5}{\sqrt{3}}$ (C) $\frac{3}{\sqrt{2}}$ (D) $3\sqrt{2}$ (E) $5\sqrt{3}$

Space for rough work

63. If the line $y = kx$ touches the parabola $y = (x-1)^2$, then the values of k are
 (A) 2, -2 (B) 0, 4 (C) 0, -2 (D) 0, 2 (E) 0, -4
64. If the semi-major axis of an ellipse is 3 and the latus rectum is $\frac{16}{9}$, then the standard equation of the ellipse is
 (A) $\frac{x^2}{9} + \frac{y^2}{8} = 1$ (B) $\frac{x^2}{8} + \frac{y^2}{9} = 1$ (C) $\frac{x^2}{9} + \frac{3y^2}{8} = 1$
 (D) $\frac{3x^2}{8} + \frac{y^2}{9} = 1$ (E) $\frac{x^2}{9} + \frac{8y^2}{3} = 1$
65. If a point $P(x, y)$ moves along the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and if C is the centre of the ellipse, then, $4 \max\{CP\} + 5 \min\{CP\} =$
 (A) 25 (B) 40 (C) 45 (D) 54 (E) 16
66. The one end of the latus rectum of the parabola $y^2 - 4x - 2y - 3 = 0$ is at
 (A) (0, -1) (B) (0, 1) (C) (0, -3) (D) (3, 0) (E) (0, 2)

Space for rough work

67. The angle between the two vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 2\hat{j} + 2\hat{k}$ is equal to
 (A) $\cos^{-1}\left(\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{6}\right)$ (C) $\cos^{-1}\left(\frac{5}{6}\right)$ (D) $\cos^{-1}\left(\frac{1}{18}\right)$ (E) $\cos^{-1}\left(\frac{1}{3}\right)$
68. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are coplanar and $|\vec{c}| = \sqrt{3}$, then
 (A) $\alpha = \sqrt{2}, \beta = 1$ (B) $\alpha = 1, \beta = \pm 1$ (C) $\alpha = \pm 1, \beta = 1$
 (D) $\alpha = \pm 1, \beta = -1$ (E) $\alpha = -1, \beta = \pm 1$
69. Let $P(1, 2, 3)$ and $Q(-1, -2, -3)$ be the two points and let O be the origin. Then $|\overrightarrow{PQ} + \overrightarrow{OP}| =$
 (A) $\sqrt{13}$ (B) $\sqrt{14}$ (C) $\sqrt{24}$ (D) $\sqrt{12}$ (E) $\sqrt{8}$
70. Let $ABCD$ be a parallelogram. If $\overrightarrow{AB} = \hat{i} + 3\hat{j} + 7\hat{k}$, $\overrightarrow{AD} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and \vec{p} is a unit vector parallel to \overrightarrow{AC} , then \vec{p} is equal to
 (A) $\frac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k})$ (B) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$ (C) $\frac{1}{7}(3\hat{i} + 6\hat{j} + 2\hat{k})$
 (D) $\frac{1}{7}(6\hat{i} + 2\hat{j} + 3\hat{k})$ (E) $\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$

Space for rough work

71. Let $\overrightarrow{OB} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\overrightarrow{OA} = 4\hat{i} + 2\hat{j} + 2\hat{k}$. The distance of the point B from the straight line passing through A and parallel to the vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ is
- (A) $\frac{7\sqrt{5}}{9}$ (B) $\frac{5\sqrt{7}}{9}$ (C) $\frac{3\sqrt{5}}{7}$ (D) $\frac{9\sqrt{5}}{7}$ (E) $\frac{9\sqrt{7}}{5}$
72. If $\vec{a} = \lambda\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + \lambda\hat{k}$ are at right angle, then the value $|\vec{a} + \vec{b}| - |\vec{a} - \vec{b}|$ is equal to
- (A) 2 (B) 1 (C) 0 (D) -1 (E) -2
73. Let the position vectors of the points A, B and C be \vec{a}, \vec{b} and \vec{c} respectively. Let Q be the point of intersection of the medians of the triangle ΔABC . Then $\overrightarrow{QA} + \overrightarrow{QB} + \overrightarrow{QC} =$
- (A) $\frac{\vec{a} + \vec{b} + \vec{c}}{2}$ (B) $2\vec{a} + \vec{b} + \vec{c}$ (C) $\vec{a} + \vec{b} + \vec{c}$ (D) $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$ (E) $\vec{0}$
74. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) $\frac{2\pi}{3}$

Space for rough work

75. The projection of the line segment joining $(2, 0, -3)$ and $(5, -1, 2)$ on a straight line whose direction ratios are $2, 4, 4$ is equal to

(A) $\frac{11}{6}$ (B) $\frac{10}{3}$ (C) $\frac{13}{3}$ (D) $\frac{13}{6}$ (E) $\frac{11}{3}$

76. The angle between the straight line $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + s(\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$ is

(A) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{2}}{6}\right)$ (C) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$
(D) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (E) $\sin^{-1}\left(\frac{2}{3}\right)$

77. If a straight line makes angles α, β, γ with the coordinate axes,

then $\frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} + \frac{1}{\sec 2\beta} - 2 \sin^2 \gamma =$

(A) -1 (B) 1 (C) -2 (D) 2 (E) 0

Space for rough work

78. The equation of the plane which bisects the line segment joining the points $(3, 2, 6)$ and $(5, 4, 8)$ and is perpendicular to the same line segment, is
- (A) $x + y + z = 16$ (B) $x + y + z = 10$ (C) $x + y + z = 12$
 (D) $x + y + z = 14$ (E) $x + y + z = 15$
79. The foot of the perpendicular from the point $(1, 6, 3)$ to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is
- (A) $(1, 3, 5)$ (B) $(-1, -1, -1)$ (C) $(2, 5, 8)$
 (D) $(-2, -3, -4)$ (E) $\left(\frac{1}{2}, 2, \frac{1}{2}\right)$
80. The plane $x + 3y + 13 = 0$ passes through the line of intersection of the planes $2x - 8y + 4z = p$ and $3x - 5y + 4z + 10 = 0$. If the plane is perpendicular to the plane $3x - y - 2z - 4 = 0$, then the value of p is equal to
- (A) 2 (B) 5 (C) 9 (D) 3 (E) -1
81. If a straight line makes the angles 60° , 45° and α with x , y and z axes respectively, then $\sin^2 \alpha =$
- (A) $\frac{3}{4}$ (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) $\frac{1}{4}$

Space for rough work

82. Let A and B be two events. Then $1 + P(A \cap B) - P(B) - P(A)$ is equal to
 (A) $P(\bar{A} \cup \bar{B})$ (B) $P(A \cap \bar{B})$ (C) $P(\bar{A} \cap B)$
 (D) $P(A \cup B)$ (E) $P(\bar{A} \cap \bar{B})$
83. The mean of five observations is 4 and their variance is $5 \cdot 2$. If three of these observations are 2, 4 and 6, then the other two observations are
 (A) 3 and 5 (B) 2 and 6 (C) 4 and 4 (D) 8 and 10 (E) 1 and 7
84. The first term of an A.P. is 148 and the common difference is -2 . If the A.M. of first n terms of the A.P. is 125, then the value of n is
 (A) 18 (B) 24 (C) 30 (D) 36 (E) 48
85. If the combined mean of two groups is $\frac{40}{3}$ and if the mean of one group with 10 observations is 15, then the mean of the other group with 8 observations is equal to
 (A) $\frac{46}{3}$ (B) $\frac{35}{4}$ (C) $\frac{45}{4}$ (D) $\frac{41}{4}$ (E) $\frac{43}{4}$
86. A function f satisfies the relation $f(n^2) = f(n) + 6$ for $n \geq 2$ and $f(2) = 8$. Then the value of $f(256)$ is
 (A) 24 (B) 26 (C) 22 (D) 28 (E) 32

Space for rough work

87. $\lim_{x \rightarrow 0} \left(\frac{10 \sin 9x}{9 \sin 10x} \right) \left(\frac{8 \sin 7x}{7 \sin 8x} \right) \left(\frac{6 \sin 5x}{5 \sin 6x} \right) \left(\frac{4 \sin 3x}{3 \sin 4x} \right) \left(\frac{\sin x}{\sin 2x} \right) =$

(A) $\frac{63}{256}$ (B) $\frac{1}{6}$ (C) $\frac{6}{5}$ (D) $\frac{1}{2}$ (E) $\frac{256}{63}$

88. The number of points at which the function $f(x) = \frac{1}{\log_e |x|}$ is discontinuous, is

(A) 1 (B) 2 (C) 3
(D) 4 (E) infinitely many

89. The value of $\lim_{y \rightarrow \infty} \left[y \sin \left(\frac{1}{y} \right) - \frac{1}{y} \right]$ is equal to

(A) 1 (B) ∞ (C) -1 (D) 0 (E) $-\infty$

90. $\lim_{x \rightarrow \infty} \left(\frac{x^2}{3x-2} - \frac{x}{3} \right) =$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{-2}{3}$ (D) $\frac{-2}{9}$ (E) $\frac{2}{9}$

Space for rough work

91. The functions f , g and h satisfy the relations $f'(x) = g(x+1)$ and $g'(x) = h(x-1)$. Then $f''(2x)$ is equal to
 (A) $h(2x)$ (B) $4h(2x)$ (C) $h(2x-1)$ (D) $h(2x+1)$ (E) $2h'(2x)$
92. If $f(x) = \sin^{-1} \left(\frac{1 - \cos 2x}{2 \sin x} \right)$, then $|f'(x)|$ is equal to
 (A) $|\sin x|$ (B) x (C) 0 (D) $|\cos x|$ (E) 1
93. If $y^2 = 100 \tan^{-1} x + 45 \sec^{-1} x + 100 \cot^{-1} x + 45 \operatorname{cosec}^{-1} x$, then $\frac{dy}{dx} =$
 (A) $\frac{x^2-1}{x^2+1}$ (B) $\frac{x^2+1}{x^2-1}$ (C) 1 (D) 0 (E) $\frac{1}{x\sqrt{x^2-1}}$
94. If $f(x) = 3x^2 - 7x + 5$, then $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x}$ is equal to
 (A) 6 (B) -7 (C) 7 (D) -6 (E) 5

Space for rough work

95. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ is equal to
 (A) $\frac{x}{\sqrt{1+x^2}}$ (B) $x\sqrt{1+x^2}$ (C) $\sqrt{1+x^2}$ (D) $\frac{1}{\sqrt{1+x^2}}$ (E) $\frac{x}{1+x^2}$
96. If $|t| < 1$, $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, then $\frac{dy}{dx} =$
 (A) $\frac{1}{x}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{x}$ (E) 1
97. If $y = \frac{x}{x+1} + \frac{x+1}{x}$, then $\frac{d^2y}{dx^2}$ at $x = 1$ is equal to
 (A) $\frac{7}{4}$ (B) $\frac{7}{8}$ (C) $\frac{1}{4}$ (D) $\frac{-7}{8}$ (E) $\frac{-7}{4}$
98. Let $f(x) = (3\sin^2(10x+11) - 7)^2$ for $x \in \mathbf{R}$. Then the maximum value of the function f , is
 (A) 9 (B) 16 (C) 49 (D) 100 (E) 121
99. If a circular plate is heated uniformly, its area expands $3c$ times as fast as its radius, then the value of c when the radius is 6 units, is
 (A) 4π (B) 2π (C) 6π (D) 3π (E) 8π

Space for rough work

100. The slope of the tangent to the curve $y = 3x^2 - 5x + 6$ at $(1, 4)$ is
 (A) -2 (B) 1 (C) 0 (D) -1 (E) 2
101. The chord joining the points $(5, 5)$ and $(11, 227)$ on the curve $y = 3x^2 - 11x - 15$ is parallel to tangent at a point on the curve. Then the abscissa of the point is
 (A) -4 (B) 4 (C) -8 (D) 8 (E) 6
102. The function $f(x) = \sin x - kx - c$, where k and c are constants, decreases always when
 (A) $k > 1$ (B) $k \geq 1$ (C) $k < 1$ (D) $k \leq 1$ (E) $k < -1$
103. If $y = m \log x + nx^2 + x$ has its extreme values at $x = 2$ and $x = 1$, then $2m + 10n =$
 (A) -1 (B) -4 (C) -2 (D) 1 (E) -3
104. If $s = 2t^3 - 6t^2 + at + 5$ is the distance travelled by a particle at time t and if the velocity is -3 when its acceleration is zero, then the value of a is
 (A) -3 (B) 3 (C) 4 (D) -4 (E) 2
105. $\int \frac{(1+x)e^x}{\cot(xe^x)} dx$ is equal to
 (A) $\log |\cos(xe^x)| + C$ (B) $\log |\cot(xe^x)| + C$ (C) $\log |\sec(xe^{-x})| + C$
 (D) $\log |\cos(xe^{-x})| + C$ (E) $\log |\sec(xe^x)| + C$

Space for rough work

106. $\int \frac{x^5}{\sqrt{1+x^3}} dx =$

- (A) $\frac{2}{9}\sqrt{1+x^2}(x^3-9)+C$ (B) $\frac{2}{9}\sqrt{x^3-9}(1+x^2)+C$ (C) $\frac{2}{9}\sqrt{1+x^3}+C$
 (D) $\frac{2}{9}\sqrt{1+x^3}(x^3-2)+C$ (E) $\frac{2}{9}\sqrt{1+x^2}(x^3+9)+C$

107. $\int \frac{4e^x}{2e^x-5e^{-x}} dx =$

- (A) $4\log|e^x-5|+C$ (B) $\frac{1}{4}\log|e^{2x}-5|+C$ (C) $\log|2e^x-5e^{-x}|+C$
 (D) $4\log|2e^x-5|+C$ (E) $\log|2e^{2x}-5|+C$

108. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx =$

- (A) $\frac{x^2}{2} + 2x + \log|x| + C$ (B) $\frac{x^2}{2} + 2 + \log|x| + C$
 (C) $\frac{x^2}{2} + x + \log|x| + C$ (D) $\frac{x^2}{2} + 2x + 2\log|x| + C$
 (E) $\frac{x^2}{2} - 2x + \log|x| + C$

109. $\int \frac{x^{n-1}}{x^{2n}+a^2} dx =$

- (A) $\frac{1}{na} \tan^{-1}\left(\frac{x^n}{a}\right) + C$ (B) $\frac{n}{a} \tan^{-1}\left(\frac{x^n}{a}\right) + C$ (C) $\frac{n}{a} \sin^{-1}\left(\frac{x^n}{a}\right) + C$
 (D) $\frac{n}{a} \cos^{-1}\left(\frac{x^n}{a}\right) + C$ (E) $\frac{1}{na} \cot^{-1}\left(\frac{x^n}{a}\right) + C$

Space for rough work

110. $\int \frac{(x+1)^2}{x(x^2+1)} dx =$

- (A) $\log|x(x^2+1)| + C$ (B) $\log|x| + C$ (C) $\log|x| + 2\tan^{-1}x + C$
 (D) $\log\left(\frac{1}{1+x^2}\right) + C$ (E) $2\log|x| + \tan^{-1}x + C$

111. $\int \frac{1}{x \log x^2} dx =$

- (A) $\frac{1}{2} \log|\log x^2| + C$ (B) $\log|\log x^2| + C$ (C) $2 \log|\log x^2| + C$
 (D) $4 \log|\log x^2| + C$ (E) $\frac{1}{4} \log|\log x^2| + C$

112. $\int_0^1 \frac{1}{(x^2+16)(x^2+25)} dx =$

- (A) $\frac{1}{5} \left[\frac{1}{4} \tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{5} \tan^{-1}\left(\frac{1}{5}\right) \right]$ (B) $\frac{1}{9} \left[\frac{1}{4} \tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{5} \tan^{-1}\left(\frac{1}{5}\right) \right]$
 (C) $\frac{1}{4} \left[\frac{1}{4} \tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{5} \tan^{-1}\left(\frac{1}{5}\right) \right]$ (D) $\frac{1}{9} \left[\frac{1}{5} \tan^{-1}\left(\frac{1}{4}\right) - \frac{1}{4} \tan^{-1}\left(\frac{1}{5}\right) \right]$
 (E) $\frac{1}{9} \left[\frac{3}{4} \tan^{-1}\left(\frac{1}{4}\right) - \frac{4}{5} \tan^{-1}\left(\frac{1}{5}\right) \right]$

Space for rough work

113. $\int_{-1}^1 x(1-x)(1+x) dx =$

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) -1 (E) 0

114. The area bounded by $y = x^2 + 3$ and $y = 2x + 3$ is (in sq. units)

- (A) $\frac{12}{7}$ (B) $\frac{4}{3}$ (C) $\frac{3}{4}$ (D) $\frac{8}{3}$ (E) $\frac{3}{8}$

115. The value of $\int_{-\pi}^{\pi} \frac{\sin^2 x}{1+7^x} dx$ is equal to

- (A) 7π (B) π (C) $\frac{\pi}{2}$ (D) 2π (E) 7π

116. $\int_0^{\sqrt{\pi}/2} 2x^3 \sin(x^2) dx =$

- (A) $\frac{1}{\sqrt{2}} \left(1 + \frac{\pi}{4} \right)$ (B) $\frac{1}{\sqrt{2}} \left(1 - \frac{\pi}{4} \right)$ (C) $\frac{1}{\sqrt{2}} \left(\frac{\pi}{2} - 1 \right)$
 (D) $\frac{1}{\sqrt{2}} \left(1 - \frac{\pi}{2} \right)$ (E) $\frac{1}{\sqrt{2}} \left(\frac{\pi}{4} - 1 \right)$

Space for rough work

117. If $x^2 + y^2 = 1$, then

- (A) $yy'' + (y')^2 + 1 = 0$ (B) $yy'' + 2(y')^2 + 1 = 0$ (C) $yy'' - 2(y')^2 + 1 = 0$
 (D) $yy'' + (y')^2 - 1 = 0$ (E) $yy'' - (2y')^2 - 1 = 0$

118. The solution of the differential equation $y'(y^2 - x) = y$ is

- (A) $y^3 - 3xy = C$ (B) $y^3 + 3xy = C$ (C) $x^3 - 3xy = C$
 (D) $y^3 - xy = C$ (E) $x^3 - xy = C$

119. The order and degree of the differential equation $\left[2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^3y}{dx^3}$ are respectively

- (A) 2 and 2 (B) 2 and 1 (C) 3 and 2
 (D) 3 and 3 (E) 2 and 4

120. The slope of a curve at any point (x, y) other than the origin, is $y + \frac{y}{x}$. Then the equation of the curve is

- (A) $y = Cxe^x$ (B) $y = x(e^x + C)$ (C) $xy = Ce^x$
 (D) $y + xe^x = C$ (E) $(y - x)e^x = C$

Space for rough work

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